Design Considerations for Asymmetric Magnetically Coupled Resonators used in Wireless Power Transfer Applications

Gunbok Lee[1,2], Benjamin H. Waters[2], Chen Shi[2], Wee Sang Park[1] and Joshua R. Smith[2]

[1]Dept. of EE, Pohang University of Science and Technology, Pohang, Gyungbuk, 790-784, Korea
[2]Dept. of EE and CSE, University of Washington, Seattle, WA, 98195, USA

***Abstract*— Magnetic resonance coupling is a widely used technique for wireless power transfer (WPT) in biomedical and consumer electronics applications. For specific applications, device size limits the overall size of the transmit and receive coils. In this work, design considerations for an asymmetrical 4-element WPT system are investigated. For either a target efficiency or a desired WPT range, the optimal coil parameters such as Q and coupling coefficient are defined and these design considerations are experimentally verified. The results can be used to design an optimal set of coils for various WPT applications.**

I. Introduction

Magnetic resonance coupling technology has been implemented with biomedical and consumer electronic applications that demand higher power levels [1]–[3] than previous wirelessly powered devices [4]. As power levels increase, efficiency and range become important design considerations because these criteria determine the required amount of power transmitted to sufficiently power a load.

Maximum efficiency over a greater range can be achieved by using a 4-element wireless power transfer (WPT) system [1], [5]. In this system, there is an overcoupled region in which efficiency is constant for a range of distances between the transmit and receive coils. The edge of the overcoupled region is modeled by the critical coupling coefficient ($k_{critical}$). There is a tradeoff between the efficiency and the range that the overcoupled region extends. In [1] this tradeoff is analyzed for a symmetrical set of magnetically coupled resonators (MCR). However, commercial WPT applications will require different form factors and coil geometries for both the transmit and receive resonators to fit the physical dimensions of the respective transmitter or receiver device.

Analyzing WPT systems in terms of coil winding geometries and the parasitic components of a winding such as the self inductance (L), capacitance (C) and resistance (R) can be complicated, and will only be applicable to a specific set of MCRs. However, many different coil geometries can achieve the same resonator Q. Similarly, a broad range of coil to coil distances and angular misalignments can have the same coupling coefficient (CC). Therefore, the general behavior of a WPT system for any set of MCRs can be accurately modeled by characterizing the respective Q and CC (k_{ij}) for each resonator.

In this work, asymmetrical 4-element WPT systems are analyzed in terms of the resonator Qs and CCs. First, the maximum operating range of a WPT system can be achieved for a desired efficiency. Second, maximum WPT efficiency can be achieved at a desired operating distance. These design considerations are validated by constructing a set of MCRs according to the proposed Q and CC for each resonator, then verifying that maximum efficiency can be achieved for a full range of distances between the MCRs.

II. Asymmetrical 4-element coil analysis

The equivalent circuit diagram of a 4-element WPT system (Fig. 1) has the transfer function shown in (3) as a function of the coil Qs and CCs where $0 \leq k_{ij} \leq 1$.

Using the techniques outlined in [1], the critical CC and the magnitude of the critical transmission coefficient ($|S_{21}|_{critical}$) are calculated from (3):

$$k_{critical} = \sqrt{k_{12}^2 Q_1 + \frac{1}{Q_2}}\sqrt{k_{34}^2 Q_4 + \frac{1}{Q_3}} \qquad (1)$$

$$|S_{21}|_{critical} = \sqrt{\frac{k_{12}^2 Q_1 Q_2}{k_{12}^2 Q_1 Q_2 + 1}}\sqrt{\frac{k_{34}^2 Q_3 Q_4}{k_{34}^2 Q_3 Q_4 + 1}} \qquad (2)$$

where $|S_{21}|_{critical}$ is the magnitude of the transmission function (S_{21}) evaluated at $k_{23} = k_{critical}$. The amount of power delivered from source to load is modeled by $|S_{21}|^2$. This indicates the power transfer efficiency from

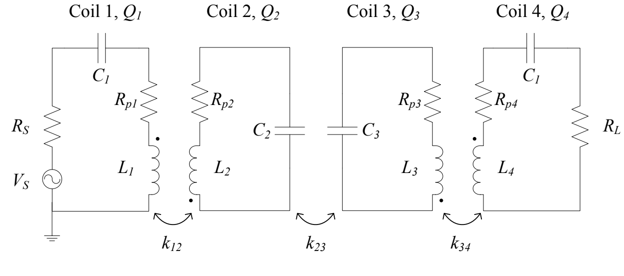

Fig. 1. Equivalent circuit model of a 4-element WPT system showing the source voltage (V_s), source resistance (R_s), load resistance (R_L), parasitic resistance (R_{pn}), inductance (L_{pn}), capacitance (C_{pn}) and quality factor (Q_n) of the n^{th} coil.

$$\left|\frac{V_L}{V_S}\right|_{\omega=\omega_0} = \sqrt{\frac{R_L}{R_S}} \frac{k_{12}k_{23}k_{34}\sqrt{Q_1 Q_4} Q_2 Q_3}{k_{12}^2 k_{34}^2 Q_1 Q_2 Q_3 Q_4 + k_{12}^2 Q_1 Q_2 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_4 + 1} \tag{3}$$

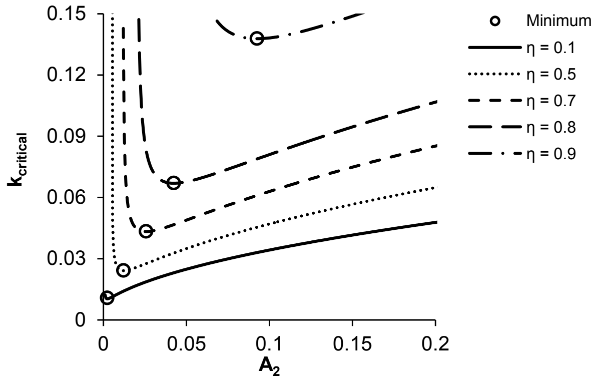

Fig. 2. $k_{critical}$ vs. A_2 in the range of η. $Q_2 = 418.6$; $Q_3 = 384.6$.

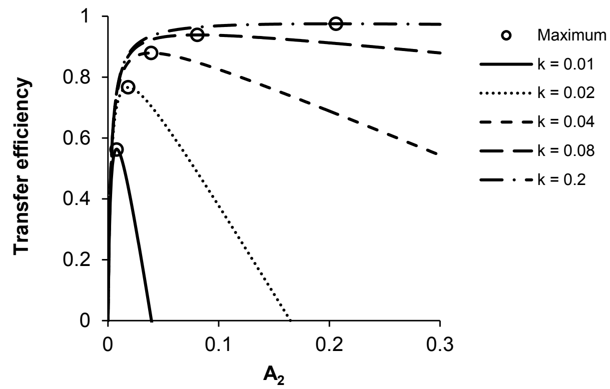

Fig. 3. Transfer efficiency vs. A_2 in the range of $k_{critical}$. $Q_2 = 418.6$; $Q_3 = 384.6$.

the first coil to the fourth coil [6]. The parameters that can be adaptively tuned in any 4-element WPT system are k_{12}, k_{34}, Q_1, and Q_4. Rewriting (1) and (2) in terms of two new parameters A_1 and A_2 that characterize all four adjustable parameters gives:

$$k_{critical}^2 = \left(A_1 + \frac{1}{Q_2}\right)\left(A_2 + \frac{1}{Q_3}\right) \tag{4}$$

$$|S_{21}|_{critical}^2 = \frac{A_1 Q_2}{A_1 Q_2 + 1} \frac{A_2 Q_3}{A_2 Q_3 + 1} \tag{5}$$

where

$$A_1 = k_{12}^2 Q_1, \quad A_2 = k_{34}^2 Q_4. \tag{6}$$

From (4) and (5), Q_2 and Q_3 should always be maximized for both maximum range and efficiency (η). By determining the required values of A_1 and A_2 to achieve a desired range or efficiency, the specific Q and CC can then be arbitrarily chosen from (6).

A. Case 1: Maximize range for a given efficiency

The value of A_1 to achieve a desired $\eta = |S_{21}|_{critical}^2$ can be found by solving (5) for A_1 as shown in (7):

$$A_1 = \frac{\eta}{Q_2} \frac{A_2 Q_3 + 1}{A_2 Q_3 (1-\eta) - \eta} \tag{7}$$

where

$$Q_4 \geq A_2 > \frac{\eta}{Q_3(1-\eta)}. \tag{8}$$

The lower bound in (8) comes from the condition that the denominator of (7) be a positive number. The upper bound in (8) is due to the maximum value of $k_{34} = 1$. Substituting (7) into (4) and solving $\frac{dk_{critical}^2}{dA_2} = 0$ yield the optimal value of A_2 at the minimum $k_{critical}$:

$$A_{2\_optimum} = \frac{\eta + \sqrt{\eta}}{Q_3(1-\eta)}. \tag{9}$$

From (7) and (9), the minimum $k_{critical}$ (maximum range) for the targeted efficiency can be calculated for a given η, Q_2 and Q_3 (Fig. 2).

B. Case 2: Maximize efficiency for a defined range

The $k_{critical}$ will be fixed by defining a targeted range for the WPT application. After calculating $k_{critical}$ based on the desired distance [7], A_1 can be defined in terms of A_2 by solving (4) for A_1 as shown in (10):

$$A_1 = \frac{k^2 Q_3}{A_2 Q_3 + 1} - \frac{1}{Q_2} \tag{10}$$

where the bounds on A_2 in (11) are defined in the same way as the bounds on A_2 in (8):

$$A_2 \leq Q_4 < k^2 Q_2 - \frac{1}{Q_3}. \tag{11}$$

Substituting (10) into (5) and solving $\frac{d|S_{21}|_{critical}^2}{dA_2} = 0$ yield the optimal value of A_2 at maximum efficiency:

$$A_{2\_optimum} = \frac{k\sqrt{Q_2 Q_3} - 1}{Q_3}. \tag{12}$$

Since $A_{2\_optimum}$ must be a positive number, (12) provides another boundary condition:

$$k > \frac{1}{\sqrt{Q_2 Q_3}} \tag{13}$$

If k doesn't satisfy (13), then it will not be possible to achieve maximum efficiency at the targeted distance. By selecting k_{12} and Q_1 to achieve the A_1 value from (10),

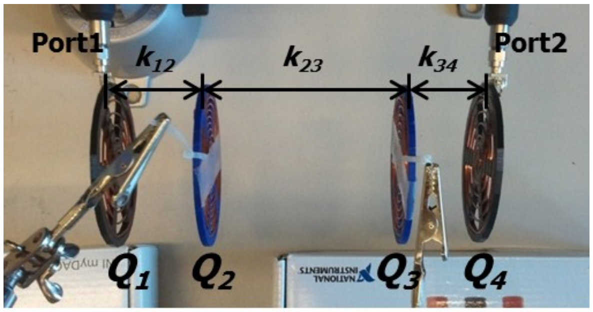

Fig. 4. Photograph of the 4-element WPT system. $Q_{1,4} = 1.9$; $Q_2 = 418.6$; $Q_3 = 384.6$. The copper wire (1.024-mm thickness) is used for coils.

and selecting k_{34} and Q_4 to achieve $A_{2\_optimum}$ from (12), maximum achievable efficiency will be reached for a given k, Q_2 and Q_3 (Fig. 3).

III. Experimental Results

To verify the equations derived in Section II, a 4-element WPT system was designed (Fig. 4). The outer and inner diameters for all coils are 700 mm and 0 mm, respectively. Coils 1 and 4 both have a 6-mm pitch and 6 turns. Coil 2 has a 3-mm pitch and 12 turns and coil 3 has a 4-mm pitch and 9 turns. All coils are tuned to 13.56 MHz using lumped capacitors. Coils 2 and 3 are held in place using clamps that have a negligible effect on wireless power transfer.

In this experimental configuration, Q_{1-4} are fixed (Fig. 4) so that the only adjustable parameters are k_{12} and k_{34}. To verify that maximum efficiency at a given distance between coil 2 and 3 is achieved, k_{12} and k_{34} were set according to the procedure outlined in Section II-B. Several distances between the second and third coils were tested from 20 mm to 130 mm using 10 mm steps. Then, k_{12} and k_{34} were determined using (6), (10) and (12). To experimentally achieve these CCs, the corresponding distances between coils $1-2$ and coils $3-4$ were set using relation between distance and CC in [7].

The efficiency at each distance was measured using the HP8753ES vector network analyzer from the extracted $|S_{21}|^2$ and compared to the expected calculated efficiencies (Fig. 5). The experimental results match the calculated efficiencies with $R^2 = 0.9637$. The errors are higher for short distances (high $A_{2\_optimum}$) because of parasitic effects such as cross coupling (k_{13}, k_{14}, and k_{24}) and capacitive feed through, which are not modeled in the theoretical transfer function [1].

IV. Conclusion

This paper provides a method to optimize coil parameters for asymmetrical 4-element MCRs used in various WPT systems. Using the derived equations for range and efficiency, the optimal coil parameters are obtained for both minimum $k_{critical}$ at a desired efficiency and maximum efficiency at a desired $k_{critical}$. The analysis characterizes the WPT system only in terms of resonator Qs and CCs so that it is easily adaptable to any size geometries, specifically coils targeted for biomedical and consumer electronic applications.

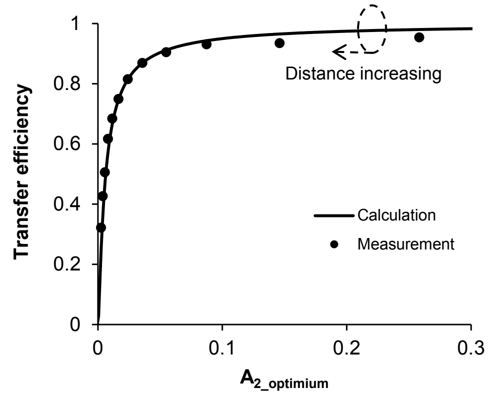

Fig. 5. Comparison result of transfer efficiency between calculation and measurement in function of $A_{2\_optimum}$ in the range of distance.

Acknowledgment

This work was supported by the Brain Korea 21 Project in 2012.

References

[1] A. Sample, D. Meyer, and J. Smith, "Analysis, experimental results, and range adaptation of magnetically coupled resonators for wireless power transfer," *IEEE Trans. Ind. Electron.*, vol. 58, no. 2, pp. 544 –554, feb. 2011.

[2] B. Waters, A. Sample, P. Bonde, and J. Smith, "Powering a ventricular assist device (vad) with the free-range resonant electrical energy delivery (free-d) system," *Proc. IEEE*, vol. 100, no. 1, pp. 138 –149, jan. 2012.

[3] Z. N. Low, R. Chinga, R. Tseng, and J. Lin, "Design and test of a high-power high-efficiency loosely coupled planar wireless power transfer system," *IEEE Trans. Ind. Electron.*, vol. 56, no. 5, pp. 1801 –1812, may 2009.

[4] J. Hirai, T.-W. Kim, and A. Kawamura, "Study on intelligent battery charging using inductive transmission of power and information," *IEEE Trans. Power Electron.*, vol. 15, no. 2, pp. 335 –345, mar 2000.

[5] B. Cannon, J. Hoburg, D. Stancil, and S. Goldstein, "Magnetic resonant coupling as a potential means for wireless power transfer to multiple small receivers," *IEEE Trans. Power Electron.*, vol. 24, no. 7, pp. 1819 –1825, july 2009.

[6] T. P. Duong and J.-W. Lee, "Experimental results of high-efficiency resonant coupling wireless power transfer using a variable coupling method," *IEEE Microw. Wireless Compon. Lett., IEEE*, vol. 21, no. 8, pp. 442 –444, aug. 2011.

[7] C. Zierhofer and E. Hochmair, "Geometric approach for coupling enhancement of magnetically coupled coils," *IEEE Trans. Biomed. Eng.*, vol. 43, no. 7, pp. 708 –714, july 1996.